

Invited Lecture

Influence of University-Based Learning Opportunities on the Professional Development of Future Mathematics Teachers

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ABSTRACT The article focuses two areas related to learning opportunities of future mathematics teachers within their university studies, namely the elementary mathematics from a higher standpoint and practical activities as part of university studies. Thereby the article refers to several comparative studies on mathematics teachers' professional competence which are shortly summarized in the beginning. Afterwards along with conceptual considerations about elementary mathematics from a higher standpoint examples for the integration of the concept in studies on teachers' professional competence as well as practical experiences from university courses are described. Subsequently empirical results from a study evaluating the professional development of future teachers in longer practical activities are depicted.

Keywords: Future mathematics teachers; Mathematics teachers' professional competence; Mathematics teacher education; Elementary mathematics from a higher standpoint; Practical activities.

1. Introduction

University-based learning opportunities constitute to a large part the professional competence of a teacher and thereby form a central basis for the continuous professional development of teachers. A core issue of university-based teacher education though of course is the imparting of theoretical competences. Alongside yet also other components, for example integrated practical experiences in school, are parts of teacher education at a university. This leads to questions of how to concretely form respective programs and thus for example to the question which contents should be included. Furthermore, with regard to the efficiency of teacher education an arising question is how to conceptualize and measure the future teachers' competences understood as an outcome of teacher education.

In the following against this general background two aspects of university-based teacher education for future mathematics teachers are discussed. The first aspect affects the question which topics should be included with regard to the subject matter knowledge, though especially focusing on the idea of elementary mathematics from a

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higher standpoint. After these considerations related to knowledge-based elements of teacher education the next section addresses the area of practical activities. Therefore, referring to the debate of the efficacy and conceptualization of respective components of university-based teacher education, some results of an empirical study are summarized. As the paragraphs commonly refer to different studies of future teachers' and practicing teachers' competences, in the first section a short summary of some central respective studies precedes.

2. Studies on Mathematics Teachers' Professional Competence

During the last around 15 years the empirical interest on the professional competence of (future) mathematics teachers remarkably raised and several respective studies have been carried out. Often, these studies aimed at both, an empirical analysis of mathematics teachers' professional competence as well as the further development of belonging theoretical frameworks. That is why we can, parallel to a constant growth of empirical results, also identify a constant development and ramification of theoretical approaches towards mathematics teachers' professional competence.

An example for these studies is the TEDS-M-study (Teacher Education and Development Study in Mathematics) (Blömeke et al., 2014; Tatto et al., 2012), an international comparative study with around 23 000 participants from 17 countries. TEDS-M indeed covered two studies, one focusing on future primary mathematics teachers and one focusing on future secondary mathematics teachers. TEDS-M examined and compared the various national policies and institutional learning opportunities and the future mathematics teachers' professional competence understanding the latter as an outcome of the respective mathematics teacher education system.

With regard to the understanding of professional competence the theoretical framework of TEDS-M takes up the understanding of competence by Weinert (2001) with its distinction between cognitive and affective-motivational aspects. Concerning the cognitive aspects, that is the future teachers' professional knowledge, TEDS-M refers to the prominent approach by Shulman (1986) and distinguishes between content knowledge, pedagogical content knowledge and general pedagogical knowledge. Concerning the affective-motivational aspects TEDS-M focuses professional motivation and self-regulation and beliefs both about mathematics as well as about teaching and learning mathematics (Döhrmann et al., 2012).

The TEDS-M study (together with its predecessor study MT21 (Mathematics Teaching in the 21st century), Schmidt et al., 2007) laid the basis for a still ongoing extensive research program covering various aspects of professional competence of mathematics teachers (Kaiser and König, 2019).

Another example of a study is the COACTIV-study (Professional competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy). COACTIV focused on practicing mathematics teachers teaching in lower secondary level. Especially, COACTIV was able to set the teachers'

data in relation to their students' achievement data gained within the German extension of PISA 2003 (Baumert et al., 2010).

With regard to its model of teachers' professional competence COACTIV also refers to the concept of competence by Weinert (2001) and distinguishes between professional knowledge, beliefs, values and goals, motivational orientations and self-regulation. Professional knowledge in particular is further divided into content knowledge, pedagogical content knowledge, pedagogical and psychological knowledge, organizational knowledge and counseling knowledge (Baumert and Kunter, 2013).

Two other studies, which are related to each other, are the "Mathematics Teaching and Learning to Teach Project" and the "Learning Mathematics for Teaching Project" (Ball et al. 2008), focusing on the work of elementary teachers. The theoretical core concept is an attempt to further develop and precise the approach by Shulman by introducing the concept of "mathematical knowledge for teaching", which is understood as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). This concept from a theoretical perspective covers the following domains assigned to the areas of subject matter knowledge and pedagogical content knowledge: common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum (for details Ball et al., 2008). From an empirical perspective in turn there is a distinction between the domains of knowledge of students and content and knowledge of content, along with a distinction of content areas (Hill et al., 2004). Also, in this project there was the possibility to analyze the relation between teachers' knowledge and students' achievement (Hill et al., 2005).

Along with the ongoing development of theoretical positions about teachers' professional competence, also the theoretical frameworks as well as the methodical approaches of the respective studies further develop. Thus, in more recent studies the understanding of professional competence often changed toward an understanding of competence as continuum (Blömeke et al., 2015). As a consequence, approaches trying to evaluate teachers' professional competence closer to reality in comparison to paper-and-pencil-tests came to the fore. A typical approach for this new kind of instruments are video-based studies. Such a study is for example the TEDS-FU-study, a follow up study to the above-mentioned TEDS-M-study. The participants of TEDS-FU all also participated in TEDS-M at the end of their teacher education and were in the fourth year of their professional practice when participating in TEDS-FU making the study a longitudinal study offering insights into the professional development of mathematics teachers in the first years of their professional practice. The theoretical framework of TEDS-FU extended the framework of TEDS-M with references to the concept of noticing (Van Es and Sherin, 2002) and the expert-novice perspective (Chi, 2011; Berliner, 2001). Departing from these concepts the theoretical framework of TEDS-FU as mentioned focuses on the idea of competence as a continuum especially referring

to the situation-specific skills, that is perception, interpretation and decision-making (PID-model) (Kaiser et al., 2015; Kaiser et al., 2017).

3. Elementary Mathematics from a Higher Standpoint

Internationally as well as often on national levels there is a large variety of different systems of teacher education. And even though the general distinction between mathematics, mathematics pedagogy and general pedagogy can serve as a possibility to classify different components of various teacher education systems, still the concrete topics covered by a certain teacher education program within the respective areas strongly differ amongst different systems (Blömeke and Kaiser, 2012). In particular with regard to content knowledge each system has to answer the question which kind of mathematics it wants the future teachers to gain competencies in. And in turn, studies on the professional competence of mathematics teachers have to answer the question which kind of mathematics they want to include into its theoretical framework and therefore measure. Both questions lead to the same core, that is the question, which mathematics teachers need for successfully teaching mathematics in school.

The last question is not a new one. One famous approach is the idea of “elementary mathematics from a higher standpoint” by Felix Klein (Klein, 2016a, 2016b, 2016c). The ideas were indeed developed more than 100 years ago, but are still often referred to, for example also in prominent positions at ICME-conferences (e.g., the lecture of the recipient of the Felix Klein Medal in 2007, Jeremy Kilpatrick, at ICME-11 (Kilpatrick, n.y.) and “The Legacy of Felix Klein” as one of the themes at the “Thematic afternoon” at ICME13 (Weigand et al., 2019)).

Klein develops his ideas against the background of overcoming the “double discontinuity” describing the situation, when a future mathematics teacher finds no relation between former school mathematics and university mathematics during his university studies and afterwards when working in school has to teach school mathematics which she or he then cannot relate to her or his university studies. That is why Klein has two aims for his lecture. “On the one hand, there is an effort to impregnate the subject matter, which the schools teach with new ideas derived from modern developments of science and in accord with modern culture. [...] On the other hand, the attempt is made to take into account, in university teaching, the needs of the school teacher.” (Klein, 2016a, p. 1). His aim is “to show you the *mutual connection between problems in the various disciplines*, these connections use not to be sufficiently considered in the specialized lecture courses, and I want more specifically to emphasize the relation of these problems to those of school mathematics” (ibid., p. 2). It is important to understand that this indeed goes along with the necessity of a sound basis of knowledge with regard to subject matter. Thus, with regard to teacher education this can be summarized to what Klein “look upon as the real goal of your academic study: the ability to draw (in ample measure) from the great body of knowledge taught to you here as vivid stimuli for your teaching” (ibid., p.2).

The idea of elementary mathematics from a higher standpoint was taken up in several theoretical considerations as well as theoretical frameworks of empirical studies. For example, in the TEDS-M study the understanding of the mathematical knowledge of the future teachers prominently integrates aspects offering connectivity to the idea of elementary mathematics from a higher standpoint as in the conceptualization of TEDS-M “a teacher’s mathematical knowledge was expected to cover from a higher and reflective level at least the mathematical content of the grades the teacher would teach. In addition, a teacher was considered to need to be able to integrate the educational content into the overall mathematical context as well as to connect the content to higher levels of education” (Döhrmann et al., 2012, p. 327f.).

Also, in the COACTIV study the respective conceptualization amongst others prominently refers to the idea of elementary mathematics from a higher standpoint (Krauss et al., 2013). Here against a theoretical background of a distinction between four types of mathematical knowledge ranging from academical to everyday knowledge they “conceptualize the CK needed for teaching as knowledge of the second type: a profound mathematical understanding of the content of the secondary school mathematics curriculum” (Baumert and Kunter, 2013, p.33).

Another example for an attempt further developing corresponding constructs of the mathematical knowledge of teachers both on a theoretical as well as an empirical level is the introduction of the concept of school-related content knowledge (Dreher et al., 2018). This knowledge is described as “conceptual mathematical CK about interrelations between academic and school mathematics” (ibid., p. 329). More concrete the following facets are distinguished: “(1) knowledge about the curricular structure and its legitimation in the sense of (meta-)mathematical reasons as well as knowledge about the interrelations between school mathematics and academic mathematics in (2) top-down and in (3) bottom-up directions” (ibid., p. 330).

Concerning university-based parts of teacher education particularly an integration of elementary mathematics from a higher standpoint into future teachers’ university courses seems to foster the students’ performance (Buchholtz and Kaiser, 2013). Furthermore, future teachers’ professional knowledge about elementary mathematics from a higher standpoint internationally obviously strongly differ (Buchholtz et al., 2013).

The preceding considerations also lead to the question, how future mathematics teachers during their university studies can be supported in overcoming the double discontinuity, besides or subsidiary to the previously summarized idea of focusing on knowledge in the sense of elementary mathematics from a higher standpoint. For this, closing this paragraph in the following two quite practical experiences are described arising from a former project for fostering future mathematics teachers for upper secondary level in the first phase of their university studies (Schwarz et al., 2013, Schwarz et al. 2014), integrated into the framework of the *Universitätskolleg* of the *Universität Hamburg*, funded by the German Federal Ministry of Education and Research.

Thus, a first import issue is the necessity of a sufficient knowledge about school mathematics when entering mathematics teacher education. Indeed, in the project the majority of students had good or very good respective proficiency, however some students also had difficulties in answering belonging questions and for example could not sufficiently calculate with fractions. It is obvious, that the requirements of a mathematical teacher education are difficult to meet for these students.

The second issue is related to the idea, that conceptualizations of competence next to cognitive aspects also contain an affective and motivational component (Weinert, 2001, Blömeke et al., 2015). Thus, as a direct consequence, it is required to not only take knowledge in the sense of cognitive aspects into consideration, when aiming at the development of future teachers' competences, but also consider these affective-motivational aspects. This for example could include offering the students opportunities for accompanied reflection on their studies and their perception of it, especially focusing on the differences between school mathematics and university mathematics, which the students of course realize and often are irritated about. In the project it was already also helpful to create institutionalized opportunities in which the students amongst each other could talk about their studies and particularly realize that they are not "alone" with the challenges arising from the differences between school and academics mathematics.

4. Phases of Practical Activities during Future Teachers' University Studies

In the preceding part, the article was orientated on aspects of professional knowledge possibly acquirable through belonging university courses. In contrast in the following part the focus is laid on practical activities as part of future teachers' university studies. Concerning these practical activities one can state that there is a large variety of different realizations in international comparison but also already on national levels concepts often differ. Furthermore, practical activities in teacher education are broadly discussed from various perspectives and also the empirical results about respective parts of university studies are multilayer and varying (e.g., König et al., 2016; Arnold et al., 2014; Besa and Büdcher, 2014; Zeichner, 2010).

The project shortly described in the following (Schwarz et al., 2020) evaluates practical activities of master students at the University of Vechta, who want to become primary mathematics teachers. During their practical activities the students continuously work in school for 18 weeks, accompanied by university seminars which are hold together by university teachers and teachers from school. The study falls back on instruments of TEDS-FU (Kaiser et al., 2017; Kaiser et al., 2015) and uses them within a pre-post-test-design with two measurement points, the first before and the second after the practical activities. The video-based tasks were open and focused basically on situation-specific skills. The data of 29 students was evaluated according to qualitative content analysis (Mayring, 2014).

The results with regard to the influence of practical activities on the professional competences of the master students mirror the respective differentness of empirical results. Separately analyzing the tasks, in all tasks there was a large proportion of students formulating answers with corresponding adequacy before and after their practical activities, yet not necessarily with corresponding foci in the answers. In some tasks furthermore, the proportion of students who were able to formulate more adequate answers was bigger than the proportion of students formulating less adequate answers after their practical activities or there even were no students of the latter group. In contrary, in other tasks, the proportion of students formulating less adequate answers at the second measurement point was bigger than the proportion of students formulating more adequate answers then. In general, students formulating more adequate answers after their practical activities then were able to more substantially respectively more often refer to relevant aspects of the video and correspondingly in contrast, students formulating less adequate answers at the second measurement point then less substantially respectively less frequently refer to relevant aspects of the video. Instead for example in these cases a stronger focus is laid on superficial aspects of the shown lessons instead of a deeper going analysis of the learning processes.

Sometimes however a less adequate answers after practical activities nevertheless also could hint at a growth of the professional competences of the students. For example, when students at the second measurement point based on their experiences in school also expect and write about quality criteria which were not included in the video and therefore the students' answers cannot meet the requirements connected to noticing aspects from the video.

Keeping in mind that the tasks partially differ concerning their maximum score, with regard to a total sum score adding the varying scores from all tasks as a very first approach to the master students' development more than 60% of the participating students indeed reached a higher score after their practical activities. This result therefore gives hints for the assumption of a usefulness of the practical activities with regard to situation-specific skills. More detailed and quantitatively based results from a bigger sample will soon be available from the project TEDS-Validation-Transfer, a project within the TEDS-research-program (Kaiser and König, 2019), which focuses on the development of professional competences of future mathematics teachers with regard to professional noticing during practical activities as part of their university studies.

5. Summary and Conclusion

Professional competence of (future) mathematics teachers is a widely discussed issue, both under a theoretical as well as an empirical perspective. The article against this general background addresses two aspects of university-based teacher education, namely the integration of elementary mathematics from a higher standpoint into teacher education courses as a more theory-related part and the integration of in-

school-activities into teacher education as a more practical-related part of teacher education.

Both areas are suitable as connecting factors for central discussions about future mathematics teachers' university-based studies as both areas exemplarily refer to different perspectives on future mathematics teachers' university-based education. The question of whether and how to integrate elementary mathematics from a higher standpoint into university courses for mathematics teachers, that is integrating an area of knowledge especially orientated on the demands of mathematics teachers, thus exemplarily refers to the perspective that university-based future mathematics teacher education is university-based education *for mathematics teachers*. This perspective emphasizes the necessity to very concretely consider what kind of competences mathematics teachers in particular need in order to offer an education truly fitting to the demands of these group of students. This of course can or even has to include further subdifferentiations, for example with regard to the teaching level.

The considerations about the efficiency of practical activities as part of university studies in turn also relate to questions of how to integrate these activities into the future teachers' university education. Thereby instead of a simple unconnected integration of practical activities into the future teachers' curriculum it is, as often realized, nearby to connect the practical activities with university-based offers such as seminars for theoretically basing and reflecting the experiences in school. Considerations like that then exemplarily refer to the perspective that university-based future mathematics teacher education is the part of teacher education *taking place at a university*. This perspective can lead to reflections of how to gain benefit for the teachers' education from the particular opportunities and strengths offered by the typical characteristics of a university education.

Hence concludingly the article refers to the idea that bringing together the perspectives of a *university-based education for mathematics teachers* offers an approach to considerations about the first phase of teacher education not only with regard to the aspects shortly affected in this article but in general with regard to various facets concerning university-based mathematics teachers' education. A deliberate combination of the two perspectives offers a framework against which respective aspects concerning future mathematics teachers' university education can be contextualized. A discussion of the two perspectives and its combination therefore can contribute to the conceptualization and the measurement of the first phase of teacher education as a basis for the professional competence of future mathematics teachers.

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